INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS, VOL. 7, 263–276 (1987)

A HYDROMETEOROLOGICAL, THREE-DIMENSIONAL MODEL OF THERMAL ENERGY RELEASES INTO ENVIRONMENTAL MEDIA

N. C. MARKATOS*

Department of Chemical Engineering, National Technical University, Athens, Greece

K. PERICLEOUS

CHAM Limited, 40 High Street, Wimbledon, London SW19 5AU. U.K.

AND

R. SIMITOVIC

Thermohydraulics Section, Electrowatt, Zurich, Switzerland.

SUMMARY

A computer model has been developed to study thermal energy releases into the environment. A typical application of the model is the study of the behaviour of cooling-tower effluent under different weather and operating conditions. The model employs the full three-dimensional transport equations describing the conservation of mass, momentum and energy. The flow is treated as single-phase and the behaviour of any droplets present is calculated indirectly. The model takes into account such hydrometeorological phenomena as the effects of humidity, wind direction and speed, density variations and the presence and precipitation of droplets. Sample results from cooling tower applications are presented and discussed.

INTRODUCTION

The problem considered and need for calculations

In general, thermal releases into the environment, such as those caused by cooling towers with their plumes, are considered rather undesirable events in the landscape and their effects are feared not only by the population, but also by some experts.[1–4] Indeed, the operation of cooling towers, and of course of any other industrial stack, has the potential to cause an adverse impact in the terrestrial ecosystem through the effects of drift.[2,3] The release of vast amounts of waste heat and steam may cause a whole range of ecological and meteorological changes in the local surroundings, which are usually populated and agricultural, the consequences of which must be thoroughly examined. Sun shading, saline mist, drift deposition, icing, interference with aircraft and interaction with other chemical stack effluents are just some of these.[5–7] The degree of injury to the environment depends upon many factors, such as salinity of source, particle or droplet size, relative humidity/dew point, precipitation, and wind speed.[3] The proper design of

0271–2091/87/030263–14$07.00

Received 24 March 1986
Revised 12 August 1986

engineering effluent dispersal devices can be done only if all these contributory factors can be estimated.

Therefore, with the increase in power of engineering equipment and machinery and the rapidly growing awareness among the population, the demands on the designer become enormous. He, and everyone involved, must be convinced that the design chosen is the optimal one, selected from the whole range of alternatives, that differ in geometrical, operational and meteorological conditions of the industrial site. The requirements for comprehensive and serious investigations of thermal or chemical releases must be met before any construction work is done.

Nature of the problem

The model will be described in the context of cooling-tower plumes, although it can also be applied to other types of plumes equally well. The physical processes governing the spread of plumes are complex and difficult to predict. The major plume parameters, such as velocity, temperature, humidity and droplet content of the effluent and of the atmosphere, geometry and power of cooling tower, and topography of the surrounding terrain, contribute, each in their own way, to the creation of the plume; but the overall effect is a complex combination of them all. Therefore the proper study of plume development cannot be done by observation of any single independent parameter alone, but must incorporate a simultaneous consideration of all processes involved. Unfortunately, such a model does not exist and it is the purpose of this work to contribute towards its development.

Previous approaches and the present contribution

In the past, attempts have been made to isolate the role of each complicated phenomenon, in order to decide which one should be modelled in more detail. These attempts try to model the simple thermodynamical and dynamical aspects in detail, and treat the complicated interactions (condensation, evaporation, diffusive and convective mixing) through a series of parametrized relationships.[2,8] This means, in practice, that the model is applied many times, each time for another parameter, and the final result derived by combining all of them. In general, existing plume and dispersion theories contain various simplifying assumptions, which limit their range of applicability. They have been reviewed in Reference 9. In that work, a model is suggested in which the concentrations in the time-averaged plume are assumed to vary in a Gaussian manner,[9,10] with dispersion coefficients given by Pasquill's stability correlations, modified to take account of terms ignored by Pasquill. The trajectory is determined by the dynamics of the plume, and this technique is typical of the existing models.[11-14] In contrast, the present approach resorts to the fully three-dimensional, time-averaged, Navier–Stokes and conservation equations governing the distributions of momentum and of scalar properties (humidity, water droplet content, temperature, etc.) to describe the plume. The effects of droplets (or particles) are accounted for indirectly, as described below.

THE MATHEMATICAL MODEL

The independent variables of the steady-state problem are the three space co-ordinates (x, y, z) of a Cartesian system. The dependent variables considered are pressure, three (mixture) velocity components (u, v, w), mixture enthalpy (h), vapour concentration (C_1) and concentration of water droplets (C_2).

The governing equations

All of the above variables are governed by differential equations of the general form

$$\operatorname{div}(\rho \mathbf{v}\Phi - \Gamma_\Phi \operatorname{grad}\Phi) = S_\Phi, \tag{1}$$

where ρ, $\mathbf{v}$, Γ_Φ and S_Φ are density, velocity vector, 'exchange coefficient' and source rate per unit volume, respectively, for variable Φ. The sources and exchange coefficients for velocities and turbulence quantities have been discussed elsewhere,[15–16] and are not repeated here. Details of the closure models specific to this work are, however, discussed in what follows.

The enthalpy (h) *equation.* The dependent variable in equation (1) is the mixture enthalpy, defined by

$$h = m_g C_{pg} T + m_l (C_{pl} T\text{-}L), \tag{2}$$

where m_g and m_l are the mass-flow rates of gas (e.g. gaseous effluent plus atmospheric air) and liquid (e.g. condensate), C_p is specific heat under constant pressure, T is temperature, L is latent heat and the subscripts g and l denote gas and liquid, respectively. Under the above definition, S_h in equation (1) is zero. Temperature is derived from

$$T = \frac{h + C_2 L}{\bar{C}_p}, \tag{3}$$

where the mixture specific heat is defined by

$$\bar{C}_p = C_1 C_{pg} + C_2 C_{pl}. \tag{4}$$

The vapour-concentration (C_1) *equation.* The source/sink terms in equation (1) for $\Phi = C_1$ are as follows:

(a) Since vapour mass is lost (in the case of condensation) or gained (in the case of evaporation), depending on atmospheric humidity, the appropriate term is

$$S_{C_1} = -M_{\text{cond}} \quad \text{or} \quad S_{C_1} = M_{\text{evap}}. \tag{5}$$

(b) The generation of droplets during condensation ('rain') necessitates the inclusion of a 'shifting' term, that accounts for the mass of vapour being lost or gained by the presence of 'rain', in a computational cell (Figure 1). In other words the condensate drifts relative to the mixture and, since the general solution refers to mixture velocities, the following term should be included:

$$S_{C_1} = \rho_1 A_n V_{\text{rain}} (C_{2,N} - C_{2,P}), \tag{6}$$

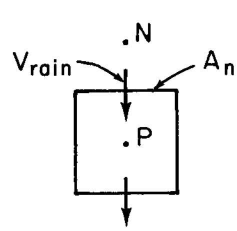

Figure 1. Condensate precipitation due to 'rain'

where A_n is cell area, V_{rain} is the droplet velocity and $C_{2,N}$ and $C_{2,P}$ are liquid concentrations at two neighbouring nodes N, P. V_{rain} can be computed from any appropriate formula, such as Stokes' law, as a function of droplet size.

The water-concentration (C_2) *equation.* This variable is important as it relates to the plume visibility.[17] The source term for this equation is

$$S_{C_2} = m_{cond} + [0, (V_{rain} - v)]\rho_l A_n(C_{2,N} - C_{2,P}), \tag{7}$$

where [ ] denotes the maximum of 0 and $(V_{rain} - v)$, v being the mixture velocity, computed from the momentum equation. The implication is that when $V_{rain} \leqslant v$ the second term vanishes (i.e. droplets remain suspended in the plume, and do not fall as 'rain'). In other words, S_{C_2} includes a term due to condensation of vapour and a term due to gain or loss of mass as water is added or removed from the cell in Figure 1, as 'rain'.

m_{cond} is a function of the local temperature, humidity, droplet size and the number of droplets per cell (determined from the solution for C_2). The most probable droplet size is also a function of the time of suspension (see details below).

The momentum equations. The only special feature is buoyancy, which is accounted for directly, without resort to empirical terms needed in the usual practice of Gaussian simulations. The source term is

$$(\rho - \rho_{ref})g\,\mathrm{Vol}, \tag{8}$$

where ρ_{ref}, g and Vol are a reference density, the gravitational acceleration and the computational-cell volume, respectively.

AUXILIARY RELATIONS

The water content, x, of fully saturated air is taken as the following function of temperature T(K):

$$T = T - 273{\cdot}16\,(^\circ\mathrm{C})$$
$$x = \begin{cases} (0{\cdot}35T + 3{\cdot}95)f, & T \leqslant 10, \\ (0{\cdot}71T + 0{\cdot}3)f, & 10 < T \leqslant 20, \\ (1{\cdot}25T - 10{\cdot}5)f, & T > 20, \end{cases} \tag{9}$$

where f accounts for height variations:

$$f = 1{\cdot}0 + 1{\cdot}3 \times 10^{-4}\,\mathrm{height}. \tag{10}$$

The mixture specific heat, C_p, is given by

$$C_p = 1005 + 1930(C_1 + C_2). \tag{11}$$

This expression is based on the following expression for the enthalpy, i, of wet air as function of water content x:

$$i = 1005\,T + x(2491 + 1{\cdot}93\,T) \times 10^3\,(\mathrm{J/kg}). \tag{12}$$

The density is the following function of enthalpy and humidity:

$$\rho = \frac{11{\cdot}7x - 4{\cdot}7 \times 10^{-6}i}{1{\cdot}005 + 1{\cdot}93x} + 1{\cdot}29\,(\mathrm{kg/m^3}). \tag{13}$$

The falling droplet terminal velocity is evaluated as follows:

$$V_{rain} = 7{\cdot}77 \times 10^{-3} R + 0{\cdot}01, \quad (14)$$

where R, the droplet radius, is computed at every z-station from the following droplet-growth formula:

$$R_z = \left[\frac{3}{4\pi\rho_l}(C_{2_z} - C_{2_{z-1}}) + R_{z-1}^3 \right]^{1/3}, \quad (15)$$

where $C_{2_{z-1}}$ and R_{z-1} are the water concentration and droplet radius, respectively, at the upstream z-station. This formula starts with an initial value, assumed to be $37{\cdot}5 \times 10^{-6}$ m, at the exit of the tower. Then, upon the assumption that the total number of droplets remain the same, the radius at a downstream station is calculated as the above function of C_2. The total number of droplets is based on the conditions at the tower exit and was evaluated to be, typically, of the order of 10^6 to 10^7.

All of the above formulae have been obtained by fitting available literature data (e.g. Reference 18 and similar). Of course, any other appropriate correlation may be implemented easily in the model.

Concerning turbulence modelling, the very simplest model was used, i.e. the constant-effective-viscosity (μ_{eff}) prescription. This involves no more than setting

$$\mu_{eff} = 0(0{\cdot}01) \times \rho \times \text{flow width} \times \text{velocity difference}. \quad (16)$$

This was done to avoid the complications (and cost) introduced by higher-order models, such as $k - \varepsilon$, which in any case are not really suitable for the present cases, where very long distances are covered by coarse computational grids.

The cooling tower shape was introduced into the Cartesian grid using the method of 'partial porosities'.[19]

SOLUTION PROCEDURE

The above set of equations was solved using the PHOENICS program[20] that involves an equation solver common to all flow problems. Details are given elsewhere.[15,20] The special features in the calculation sequence for the present work are the evaluation of sources and point-by-point variation in properties. The sequence of those special calculation steps, within a general hydrodynamic solution loop, is as follows:

(a) Calculate density from equation (13).
(b) Calculate C_p from equation (11).
(c) Calculate temperature from equation (3).
(d) Calculate saturation vapour mass fraction, x, from equation (9), accounting for height variations using equation (10).
(e) Calculate condensate mass fraction as $C_2 - x$.
(f) Calculate the new droplet radius (i.e. the radius at the current solution station) from equation (15). It is worth mentioning that the condensate is assumed to merge with existing water droplets rather than create new ones.
(g) Calculate terminal droplet velocity, from equation (14). Compare its magnitude with the mixture velocity, to determine whether the droplets are falling or remain suspended in the plume (equation (7)).
(h) Formulate relevant source terms, and add them to the appropriate equations.

Steps (a) to (h) are repeated until errors in the conservation equations diminish to a value less than 1 per cent of the total inlet flux. At this stage convergence is reached and further iterations produce virtually no change in the solution.

RESULTS AND DISCUSSION

The physical problem considered for demonstration refers to the plume of a 2000 MW cooling tower, of height 150 m and exit diameter 80 m. Many different situations were simulated corresponding to different weather conditions, ranging from typical extremely bad winter atmospheric conditions (100 per cent humidity, freezing temperature, medium strong wind, inverse vertical temperature profile) to typical optimal summer conditions. The conditions of environment and effluent considered are given in Tables I and II.

Boundary conditions and computational details

The whole integration domain extends to a maximum of 10 km in the axial, z, direction and to 3–5 km in the other two directions, typically covering a volume of 120 km^3. The grid used was $(x, y, z) = 12 \times 24 \times 38$. The boundary conditions are as follows:

(a) At the 'inlet' of the computational domain, specified wind velocity, temperature and humidity were used. The droplet concentration was set to zero.
(b) The tower was located, typically, 300 m away from the inlet. At the tower exit, specified mass and enthalpy fluxes, velocities, vapour and droplet concentrations were prescribed, as shown in Table II.
(c) At all other 'free' boundaries, atmospheric pressure was assumed to prevail, a condition that allows the calculation of the correct inflow/outflow at these boundaries, from overall continuity considerations.

Convergence was monotonic and easy to obtain, requiring only around 60 solution sweeps over the domain. The computer time required for a typical run was 1h cpu on a Perkin-Elmer 3250 mini-computer.

No grid-dependency studies were performed and therefore the presented results should be treated only qualitatively.

Results

Owing to space restrictions only a sample of the results obtained is presented here, in Figures 2–10.

Figures 2 to 5 present results that pertain to the case favourable to the formation of a long vapour plume, i.e. high wind speed, high humidity and temperatures close to freezing (winter conditions).

Table I. Environment

Conditions	Wind velocity, $\mathrm{m\,s^{-1}}$	Temperatures °C	Humidity %	Humidity $\mathrm{g\,kg^{-1}}$	Enthalpy, i $\times 10^3\,\mathrm{J\,kg^{-1}}$	Density, ρ $\mathrm{kg\,m^{-3}}$
Winter	6	−1	100	3·6	7·95	1·30
Summer	6	20	75	10·9	47·7	1·20

Table II. Effluent of cooling tower (CT)

Conditions	Load of CT,	Assisted by dry CT	Vertical velocity,	Temperature,	Throughput air	Throughput air	Density,	Vapour output,	Effluent humidity,			Effluent enthalpy
	%		ms^{-1}	°C	$\times 10^3$, m^3s^{-1}	$\times 10^3$, kgs^{-1}	kgm^{-3}	kgs^{-1}	%	gkg^{-1}	Incl. droplet gkg^{-1}	$\times 10^3$; $J kg^{-1}$
Winter	100	No	5·3	24·7	23·9	28·0	1·12	568	100	19·7	20·1	74·8
		Yes	6·2	24·8	28·1/28·9	33·0	1·17	564	81·3	16·0	17·1/16·7	65·5
	50	No	4·3	16·8	19·6	23·7	1·21	282	100	11·6	12·3	46·1
Summer	100	No	4·3	35·6	19·7	22·1	1·12	834	100	37·8	37·7	132·5

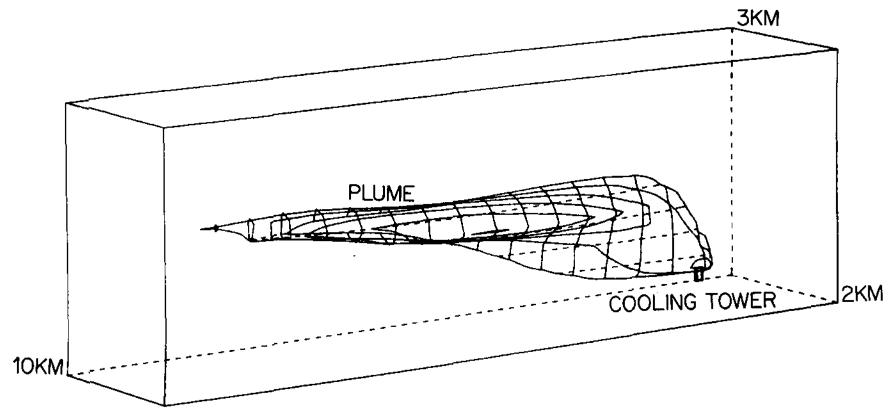

Figure 2. Droplet concentration surface, winter conditions

Figure 2 shows a contour surface of droplet concentration at a value of 5×10^{-6} kg/kg. The concentration increases inside this surface and decreases outside. The extent of the calculation domain is also shown in the Figure; the cooling tower lies on a symmetry plane. The following plume characteristics are observed:

(a) A maximum height of 1·5 km is reached at a region close to the cooling tower.
(b) The prevailing wind leads to a maximum plume length of about 8 km.
(c) The plotted surface appears much fuller than one would expect and continuous. This is because the intermittency characteristics of turbulence are not taken into account in the turbulence model used and also the results are for steady-state conditions equivalent to a time averaged plume, such as one would obtain using long exposure photography. In addition, the contour value chosen (quite arbitrarily) is not necessarily the critical value from the visibility point of view.

Nevertheless, (a) and (b) agree well with observations as seen in Reference 21 for similar atmospheric conditions.

A plume trajectory formula as given by (see, for example, Reference 22) for neutral atmosphere, predicts a rise of 1250 m at 1000 m downstream and at 5000 m a rise of 3560 m. The faster rise predicted by the present model is due to heat generated by condensation in regions of high droplet concentration, whereas further downstream evaporation becomes dominant with a resulting levelling off not accounted for by the simpler formula of Briggs.

Figure 3 shows a contour map of vapour-concentration and velocity fields in the vicinity of the tower. Contour levels are between 0·004 and 0·006 at ten regular intervals. The maximum concentration surrounds the tower, as expected.

Figure 4 shows a perspective view of vapour-concentration profiles at three stations downstream of the tower. Maximum concentration at each station occurs at the symmetry plane and decreases in magnitude downstream as the plume diffuses and also due to precipitation. The locus of the maximum defines the plume trajectory.

Figure 5 shows a temperature map at the symmetry plane. Ambient temperature is at -1°C and the plume temperature at the tower exit is 9·6°C. Contours show diminishing temperatures at 0·1°C intervals between -1 and $+1$.

Figure 6 corresponds to the winter conditions, but with 50 per cent humidity. Comparison of this Figure with Figure 2 reveals that the visible plume is of nearly the same length, i.e. 8 km, but it is thinner than the plume developed under 100 per cent humidity. This is as expected since the

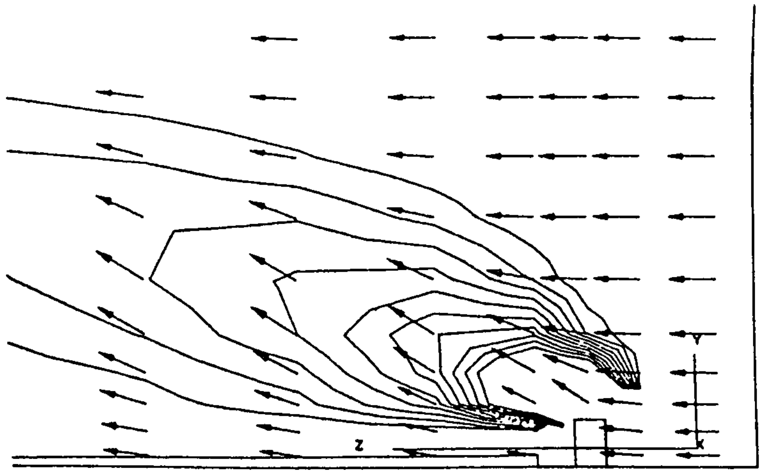

Figure 3. Vapour contours and velocity field close to the tower exit, winter conditions

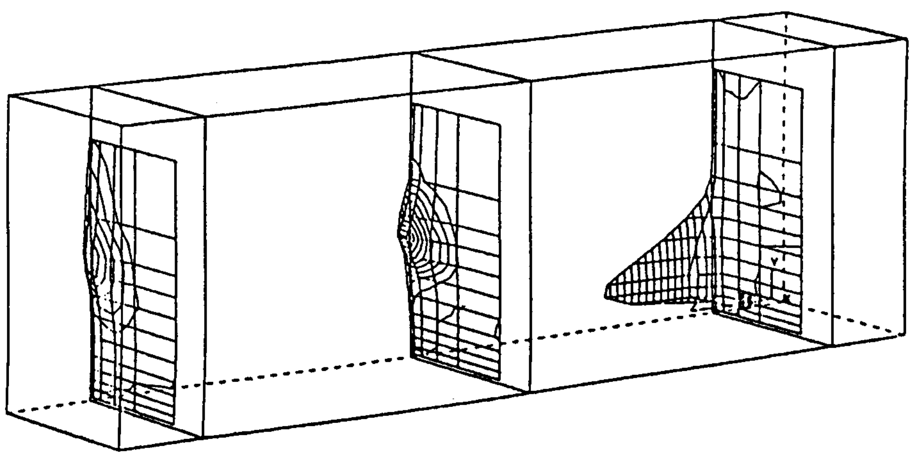

Figure 4. Droplet concentration profiles, winter conditions

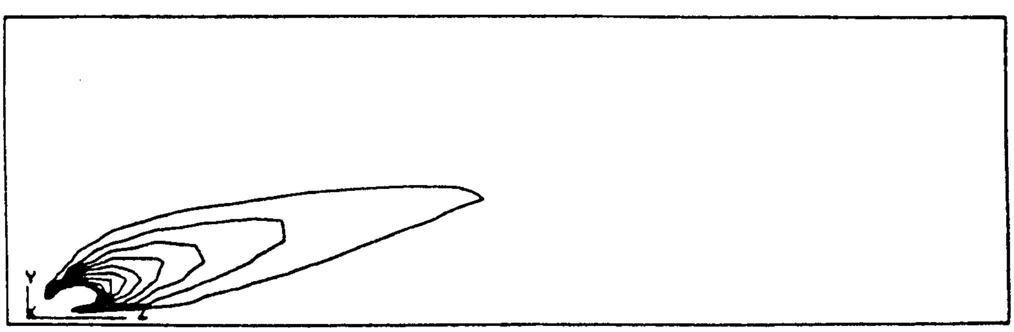

Figure 5. Temperature contours, winter conditions: $-1, (0{\cdot}1), 1°C$

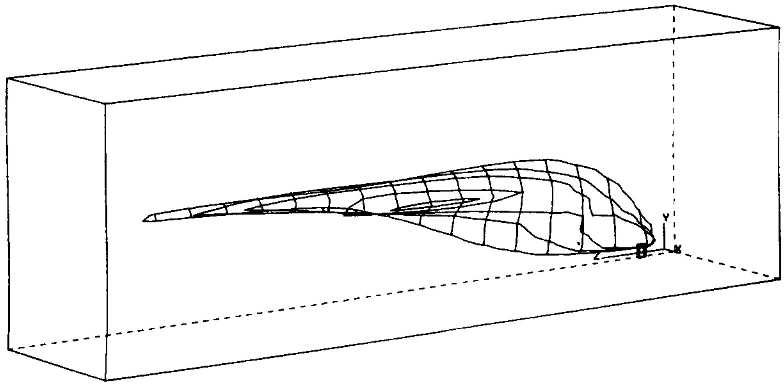

Figure 6. Droplet concentration surface, winter conditions, 50 per cent humidity

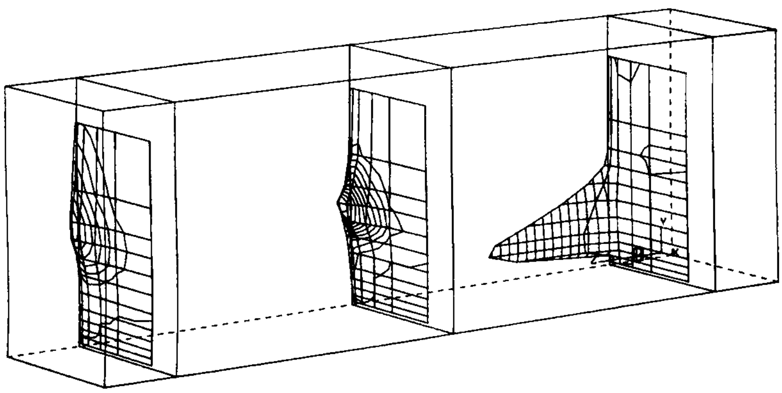

Figure 7. Droplet concentration profiles, winter conditions, 100 per cent humidity, high cooling tower load

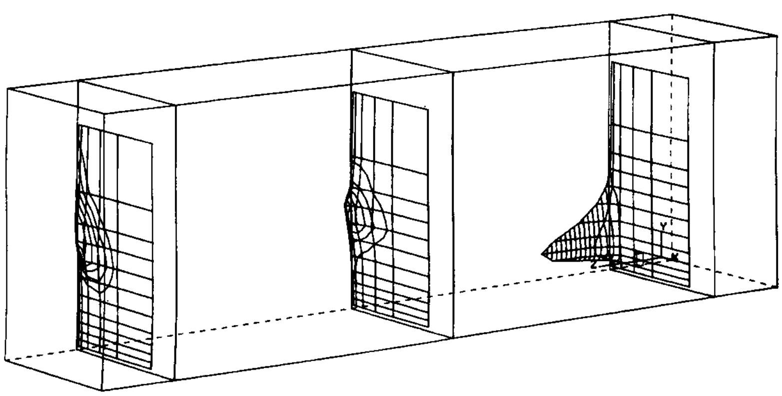

Figure 8. Droplet concentration profiles, winter conditions, 100 per cent humidity, low cooling tower load

droplets evaporate easier in a less humid atmosphere, especially close to the plume boundary.

Figures 7 and 8 present droplet profiles for the same winter conditions, but different vapour outputs from the cooling tower, representing low and high load conditions. The profiles are generally similar in shape, although the maximum value for the high load case (Figure 7) is approximately twice that of the low load case. The profiles are presented at three distances from the tower, namely $z = 1$, 5 and 9 km. The maximum excursion on Figure 7 is 10^{-3} (i.e. one gram per kilogram).

Figure 9 presents droplet section profiles at four downstream stations, for the winter conditions. The reference value plotted is 10^{-6} and the maximum value is $5{\cdot}0 \times 10^{-6}$. The locus of maxima defines the plume trajectory. Finally, Figure 10 presents a map of droplets through the symmetry plane, for summer conditions. Ten equally spaced contours are presented for values between $5{\cdot}0 \times 10^{-5}$ and $5{\cdot}0 \times 10^{-6}$.

Figures 11 and 12 refer to results obtained using an integral plume model and a Gaussian model, based on Reference 10. The former is used to trace the plume trajectory and the latter to predict the vapour concentration.

The plume trajectory coincides with the locus of maximum vapour concentration in the downstream direction. Again evaporation or condensation is not taken into account. The calculated trajectory is lower than that predicted by the present method, e.g. 500 m, 750 m and 1450 m, at 1 km, 5 km and 9 km, respectively. Vapour concentration values are of the same order as those predicted by PHOENICS, reaching a maximum of 7·5 g/kg at 1 km as opposed to 6 g/kg.

Inspection of all results obtained leads to the following main findings:

1. The cooling-tower output affects the droplet spread. Although the length of the visible part of the plume and its elevation do not change (under the same environmental conditions) the droplets spread more in the external direction, for the higher tower output. It also appears that for the lower power the plume tapers off faster.
2. The summer visible plume is considerably shorter than the winter plume (i.e. 5 km as opposed to 8 km), and it is thinner near the tower. This is because in winter the droplets tend to remain

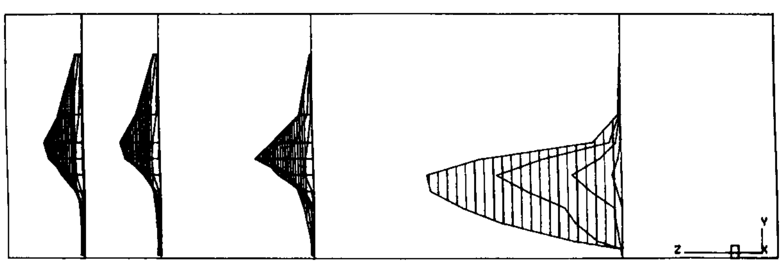

Figure 9. Droplet profiles at four downstream stations, winter conditions, 100 per cent humidity

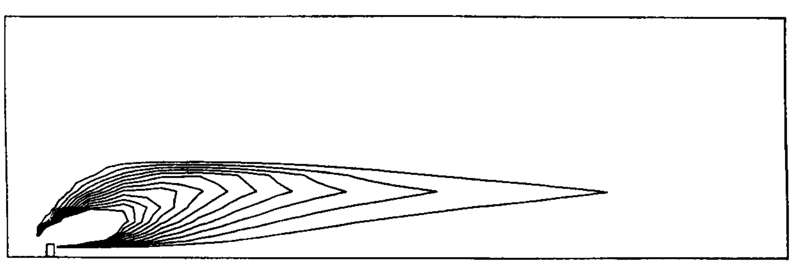

Figure 10. Map of droplets through the symmetry plane summer conditions: 5×10^{-6}, ($4{\cdot}5 \times 10^{-6}$), 5×10^{-5} kg/kg

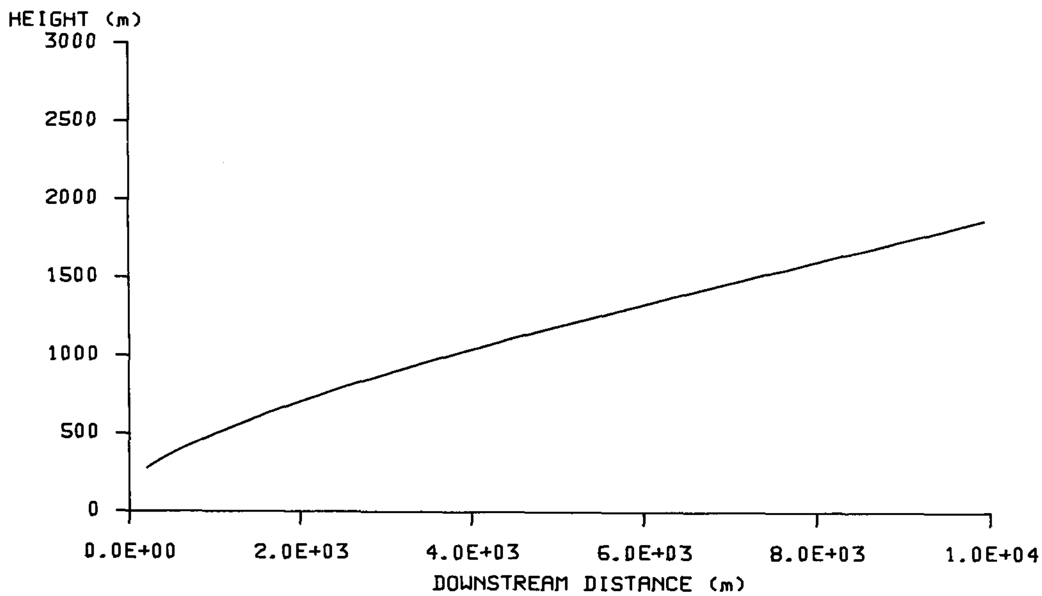

Figure 11. Plume trajectory using an integral method: outlet temperature = 282·6 K

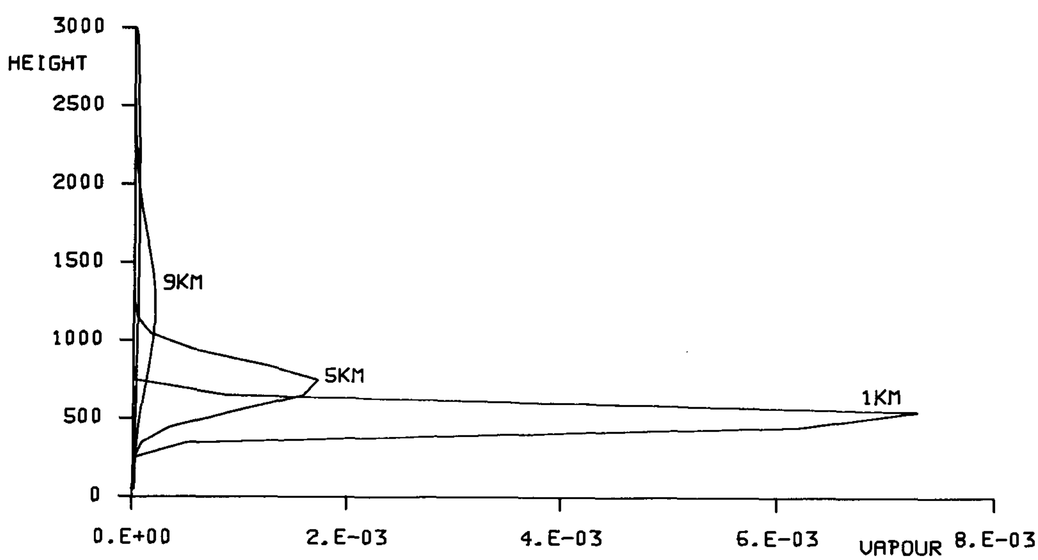

Figure 12. Vapour concentration at three downstream positions: Gaussian model

suspended due to high humidity, and buoyancy induced due to condensation.

3. It should be mentioned that the smooth boundaries of the plume in the presented Figures are obtained by artistic licence. In actual turbulent conditions the thin parts of the plume would break up and would not be visible. This intermittent nature of the plume can be simulated

only via more sophisticated (and more expensive) models such as the 'two-fluid' model of turbulence.[23,24,25]

4. Comparison with traditional statistical or integral methods indicates the same general trends of plume behaviour, but significant differences in actual trajectory and vapour concentration. This is not unexpected, considering the simplifications implied in these methods, but also one must recognize that the results obtained here are for a fairly coarse solution grid. The basis of the model has been laid and further calculations are in progress to enhance its usability and provide a better quantitative assessment.

CLOSURE

A fully three-dimensional mathematical model for studying thermal energy releases into the environment has been described, and its application to cooling-tower plumes presented. The most important advantage of the present approach is its flexibility in accepting any given conditions; most previous models are not deemed sufficiently detailed to cover, for example, all possible weather conditons. Thus, both buoyant plumes and plumes under temperature-inversion conditions can be simulated with equal ease, as well as the interactions with short or tall stacks. The model covers the case of a dense plume which falls to the ground, a case important to accidental releases of hazardous vapours. The time-mean edges of the plume and its elevation are properly calculated, as they become diffusive when the plume mixes with the surrounding air; previous methods obtained them, in general, by rules of thumb and empirical methods, and the same is true for its vertical and cross-wind directions.

The demonstration results obtained are qualitatively realistic, as was confirmed by visual observations, and give a sound insight into the nature of the problems studied.

Of course there are still many uncertainties related to the model, e.g. turbulence and appropriate auxiliary correlations. Even finding the correct input wind speed can prove a problem, because the height at which it is measured is often missing from published observations of plume rise.[9] Therefore, the proposed model is primarily intended as a framework, to be used for parametric studies that can shed light on the behaviour of uncertain parameters, in a relative manner.

Under strongly turbulent conditions plumes can break up into discrete 'puffs'. This behaviour cannot be predicted by the present model; intermittent phenomena would require more sophisticated turbulence modelling, like for example the two-fluid models,[23,24,25] that are just coming into being.

REFERENCES

1. A. Murty Kanury, 'Environmental aspects, atmospheric and aquatic dispersion of heat, geothermal heat transfer and management of process byproducts', *Mechanical Engineering*, p. 28 (1983).
2. 'Potential environmental modifications produced by large evaporative cooling towers', *Water Pollution Control Research Series, 16130 DNH 01/71*, Environmental Protection Agency. Water Quality Office.
3. 'Cooling Tower Environment—1978', *Proceedings of a Symposium on Environmental Effects of Cooling Tower Emission, WRRC Special Report No. 9*, May 1978.
4. 'Cooling Tower Environment—1978', *Proceedings of a Symposium on Environmental Effects of Cooling Tower Emission, WRRC Special Report No. 9, Supplement*, May 1978.
5. S. R. Hanna, 'Hydrometeorological aspects of electric power production', Environmental Research Laboratories, Oak Ridge, Tennessee, *ATDL Contribution File No 76/23*, December 1976.
6. S. R. Hanna, K. S. Rao and R. P. Hosker, 'Report on ATDL research on meteorological effect of thermal releases—August 1 1976–September 30 1977', Environmental Research Laboratories, Oak Ridge, Tennessee, *ATDL Contribution File No 77/27*, November 1977.
7. S. R. Hanna, 'Atmospheric effects of energy generation', Environmental Research Laboratories, Oak Ridge, Tennessee, *ATDL Contribution File No 77/9*, June 1978.

8. A. I. Weinstein and L. G. Davis, 'A parameterised numerical model of cumulus convection', Department of Meteorology, The Pennsylvania State University, Pennsylvania, May 1968.
9. D. G. Shipley, 'A mathematical model of effluent plumes', *Inst. Mech. E. Paper C96/83*, pp. 57–63 (1983).
10. K. T. Spillane and C. C. Elsum, 'Convective knockdown of cooling tower plumes', *Atmospheric Environment*, **17** (2), 227–233 (1983).
11. F. Gassman, D. Haschke and W. Solfrian, 'Mathematische Simulation von Kuhlturmfahnen und Bedeutung von Messungen', *EIR-Bericht No. 295*, Wurenlingen, 1976.
12. K. Nester, 'Modellmassige Unterschung der Meteorologischen, Auswirkungen Konzentrierter Warmequellen', *Simulation von Kuhlturmfahnen Abwarmeprojekt oberrheingebiet* (Germany), Umweltbundesamt, Kernforschungszentrum, Karlsruhe, HS/M, 1980.
13. K. Nester, 'Die Ausbreitung von Tropfchen aus Naturzugkuhlturmen bei Besonderen Meteorologischen Bedingungen', *Seminar der VdTUV*, Tropfenemission Koln, November 1980.
14. K. Nester and H. Verenkotte, 'Dynamic interaction of cooling tower and stack plumes', *Proceedings of the 13th International Technical Meeting on Air Pollution Modelling and its Application*, France, September 1982.
15. D. B. Spalding, 'Mathematical modelling of fluid mechanics, heat transfer and chemical-reaction processes', *CFDU Report No HTS/80/1*, Imperial College, London 1980.
16. N. C. Markatos and K. A. Pericleous, 'An investigation of three-dimensional fires in enclosures', *Fire Dynamics and Heat Transfer—HTD—ASME*, **25**, 115–124 (1984).
17. P. Brog, W. Hofmann, H. Sutter, H. M. von Euw and H. Fuchs, 'Some developments in plume and drift deposition modelling since 1974', Motor-Columbus, Baden, Switzerland.
18. *Leistungversuche an Kuhlturmen* (V DI-Kuhlturmregein), Deutsche Normen, DIN, June 1959.
19. A. Moult, D. B. Spalding and N. C. Markatos, 'The solution of flow problems in highly irregular domains by the finite difference method', *Trans. Inst., Chem Engr.*, **57**, 200–204 (1979).
20. D. B. Spalding, 'A general purpose computer program for multi-dimensional, one- and two-phase flow', *Mathematics and Computers in Simulation*, **XXIII**, 267–276 (1981).
21. D. Haschke, J. P. Trepp and F. Gassmann, 'Swiss realtime observations of cooling tower plume behaviour and impact', *Nuclear Europe*, 1982.
22. J. A. Jones, 'The fifth report of a working group on atmospheric dispersion', National Radiological Protection Board, *NRPB-R157*, Oxon, 1983.
23. D. B. Spalding, 'Chemical reaction in turbulent fluids', *J. Physicochemical Hydrodynamics*, **4**(4), 323 (1983).
24. M. R. Malin and D. B. Spalding, 'A two-fluid model of turbulence and its application to heated plane jets and wakes', *J. Physicochemical Hydrodynamics*, **5**(5/6), 339–362 (1984).
25. N. C. Markatos and K. A. Pericleous, 'A two-fluid model of turbulence applied to simulation of fires', *Proc. 11th IMACS World Cong.*, Norway, August 1985, Vol. 2, pp. 189–193.